Code No: 132AB **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, September - 2023

MATHEMATICS – II

(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Find the Laplace transform of the function $f(t) = t^2$. [2]

b) Find the Laplace transform of $\left(\frac{sin3t}{t}\right)$. [3]

c) Find the value of $\Gamma\left(-\frac{1}{2}\right)$. [2]

d) Express $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ in terms of Beta functions. [3]

e) Evaluate $\int_0^{\pi}\int_0^{a sin\theta} r dr d\theta$. What is the region of integration? [2]

f) Find the area between two curves $y = x; y = x^2$. [3]

g) Define Solenoidal and Irrotational of a vector. [2]

h) Find the angle between two surfaces $x^2 + y^2 + z^2 = 9$; $x^2 + y^2 - z - 3 = 0$ at (2,-1,2). [3]

i) Find the work done in moving a particle by the force field $\bar{F} = 3x^2\bar{i} + \bar{j} + z\bar{k}$ along the straight line from (0,0,0) to (2,1,3). [2]

j) If $\bar{F} = (2x^2 - 3z)\bar{i} - 2xy\bar{j} - 4x\bar{k}$ then evaluate $\int(\nabla \times \bar{F})\, dv$ over v, where v is the closed region bounded by $x = 0, y = 0, z = 0, 2x + 2y + z = 4$. [3]

PART - B

(50 Marks)

2.a) Use Laplace transform to prove that $\int_0^{\infty}\frac{sint}{t}dt = \frac{\pi}{4}$.

b) Find $L^{-1}\left[\frac{s}{(s+1)^2(s^2+1)}\right]$. [5+5]

OR

3. Solve by using Laplace transforms $y''(t) + 5y'(t) + 6y(t) = t$ with $y(0) = 1$ and $y'(0) = 1$. [10]

4.a) Evaluate $\int_0^{2a} x^3(2ax - x^2)^{3/2}\, dx$.

b) Evaluate $\int_0^{\infty} \sin x^2 dx$. [5+5]

OR

5.a) Show that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = \beta(m, n)$.

b) Express $\int_0^{\frac{\pi}{2}} sin^5\theta\, cos^{\frac{7}{2}}\theta\, d\theta$ in terms of Beta and Gamma functions and hence evaluate. [5+5]

6. Change the order of integration and hence evaluate the double integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx.$ [10]

OR

7. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ using triple integration. [10]

8.a) Prove that $\nabla(r^n) = nr^{n-2}\bar{r}$.

b) Find constants a, b, c so that the vector, $\bar{A} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational. Also find φ such that $\bar{A} = \nabla\varphi$. [5+5]

OR

9.a) Prove that $\nabla\left(\nabla.\frac{\bar{r}}{r}\right) = -\frac{2}{r^3}\bar{r}$.

b) Prove that $\nabla^2(r^n) = nr^{n-2}(n + 1)$. [5+5]

10. Verify Gauss Divergence theorem for $\bar{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ taken over the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$. [10]

OR

11. Verify Stokes theorem for $\bar{F} = (y - z + 2)\bar{i} + (yz + 4)\bar{j} - xz\bar{k}$ where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy-plane. [10]

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